Modeling Camp - Recycling

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1 Motivation

Nowadays, recycling has become a trend-topic in most of first world's nations. As a matter of fact, several materials can be recycled and there are plenty of benefits, such as reduced consumption of fresh raw materials, reduced energy usage and reduced landfills. Nevertheless, recycling comes with its own costs, such as the collection of items and transfer to the recycling center as well as sorting. Several questions naturally arise, among others

- Is recycling worth it compared to sending everything to landfills?
- Can we also obtain profit from unsorted waste sources?
- How expensive is recycling if we want to fulfill certain quotas?
- Do capacities represent a bottle-neck for the recycling process?
- Is it feasible to remove or build facilities?

In order to answer these questions we are going to derive a mathematical model for the cost-benefit of recycling. Afterwards we will try to see if optimality can be defined in any sense.

2 Mathematical model

Our first approach was in fact trying to find general information about the topic mentioned above and later on, we found an existing paper on the cost-benefit analysis of recycling, which can be found at [1]. Inspired by this paper, we have proceeded as follows.

2.1 Formulation of the model

First off, we have extracted several ideas from the paper such as what costs are involved, minimizing over the material flow and discrete-time behavior of the system. From now on the time space is going to be $\mathbb{T} := \{0, \Delta t, \dots, T\}$. A good way of picturing the problem is by thinking of several facilities interacting and exchanging materials among them. Figure 1 shows a rough idea of the problem.

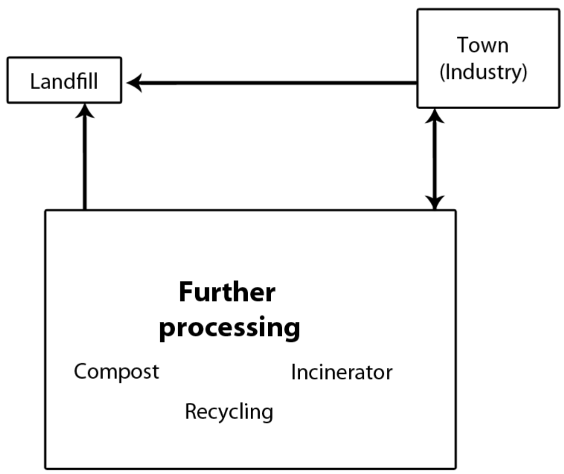


Figure 1: A simple sketch of the structure of our problem.

Once we had understood the structure of our problem, we have constructed different models and gone for one of them in particular, which has the following cost function

$$Z = \sum_t \sum_i \left(FC_i(u_i(t), t) + \sum_j TC_{ij}\left(u_{ij}(t), t\right) \right), \tag{1}$$

where $FC_i : \mathbb{R}^n \times \mathbb{T} \to \mathbb{R}$ are the facility costs of the facility i and $TC_{ij} : \mathbb{R}^n \times \mathbb{T} \to \mathbb{R}$ are the transportation costs from facility i to facility j. The former functions depend on a material vector $u \in \mathbb{R}^n$ and the time t. We inserted special vectors

$u_{ij}(t) =$ materials transported from facility i to j at time t,
$u_i(t) =$ materials available at facility i at time t after transportation;

In fact, $u_i(t)$ is constructed from $u_{ij}(t)$ by using the recursion

$$u_i(t) = \sum_j u_{ji}(t) - \sum_j u_{ij}(t) + PF_i(u_i(t - \Delta t), t - \Delta t)$$

which is the difference of the incoming and outgoing material flow adding the previously processed materials at the facility. We assume some initial values which are added instead of the previously processed materials at $t = 0$. Here $PF_i : \mathbb{R}^n \times \mathbb{T} \to \mathbb{R}^n$ describes how materials are processed, i.e. converted, sold, stored or delivered, at the facility i at time t. The processing functions should be closely related to the facility cost functions when applying this model to an example.

Natural constraints come up, namely,

$$u_i(t) \geq 0,$$
$$u_{ij}(t) \geq 0,$$

as well as upper bounds for all $u_i(t)$ and $u_{ij}(t)$ either component wise or for the total amount given by capacities. Other constraints such as mass-balance or further facility specific constraints are governed by the processing functions.

Now our problem consists in minimizing the costs Z in order to find the optimal material flow subject to the constraints mentioned above.

2.2 Assumptions and simplifications

A few assumptions have been required in order to give sense to the model, i.e.

- Sorted materials.
- Each facility handles all materials. (Can be avoided by setting corresponding capacities to zero.)
- Fixed number and location of facilities.

- Transportation takes place in one time-step.

For a material to go through its whole processing, it might take several time-steps depending on the amount of processing steps. This can be quite restrictive as some procedures usually require different times. The first simplification we have come up with and a nice way to describe a model structured as ours are processing matrices,

$$PF(u,t) := A(t)u + b(t),$$

and affine cost functions

$$FC(u,t) := c(t)^T u + \lambda(t),$$
$$TC(u,t) := d(t)^T u + \mu(t).$$

A further simplification could be to assume time-independent functions, since many features of facilities stay the same over the planning period.

2.3 Solution theory

In the most general case, we at least require the model functions to be continuous in u for each $t \in \mathbb{T}$. Since we have prescribed upper and lower bounds for our set of variables u_{ij}, the domain of Z is compact. As long as it is not empty, by Weierstraß we can guarantee the existence of optimal solutions to our minimization problem.

For the affine case we have a numerical tool, namely the SIMPLEX algorithm which provides us with a numerical solution to the problem.

2.4 Sorting extension

The sorting process consists in separating some materials from unsorted waste. Figure 2 shows schematically how we have included the sorting process.

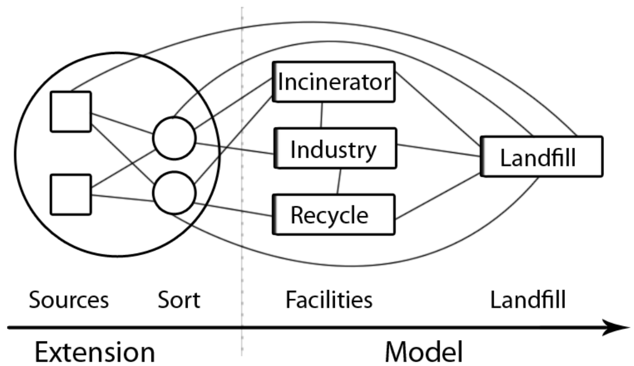


Figure 2: Sketch of the sorting extension.

The sorting extension is a pre-step to our first model, where we sort the unprocessed waste extracting a particular branch of materials and sending the rest right away to the landfills. The sources produce unsorted waste with a known decomposition. The unsorted waste is transported either to sorting facilities or to landfills. The sorting facilities sort out a specific branch of materials which are then fed into processing facilities of our previous model.

3 Simulation

We have implemented the linearized sorting extension using a flexible and modular Python program that generates an objective function and constraints from the geometry and parameters of our model. The resulting linear optimization task is fed into the LP solver PuLP. Afterwards, a visualization of the optimized waste flows in the plane is drawn in SVG files.

For technical details regarding the simulation, we encourage the reader to have a look at the Appendix A. The full source code is available under `https://github.com/damast93/recycling`.

3.1 Example Case

We will now give a qualitative interpretation of our simulation results in an example case. In order to use somewhat realistic parameters, we obtained data from various sources [Quellen einfÃ$\frac{1}{4}$gen!!], though the thorough investigation of these pieces of data was not the focus of our project.

The distances and the amount of waste we model are on the level of a US county, the planning period is 5 years. There are

- three waste sources, depicted with a W
- sorting facilities S1 and S2 with squares
- incinerators I1 and I2, a composting facility C, plastic recycling R and glass recycling G, all with a diamond symbol
- two landfills (circles)

The waste flows are visualized as the colored lines where black is unsorted, red is plastic, gray is glass, blue is unusable residues and green is biological waste. The thickness of the lines corresponds to the amount of waste sent.

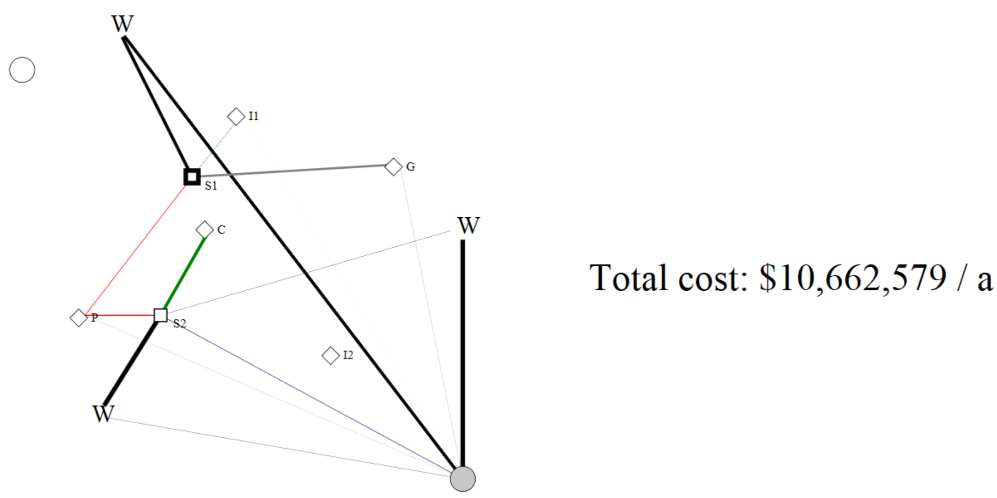


Figure 3: Simulation, $t = 0$

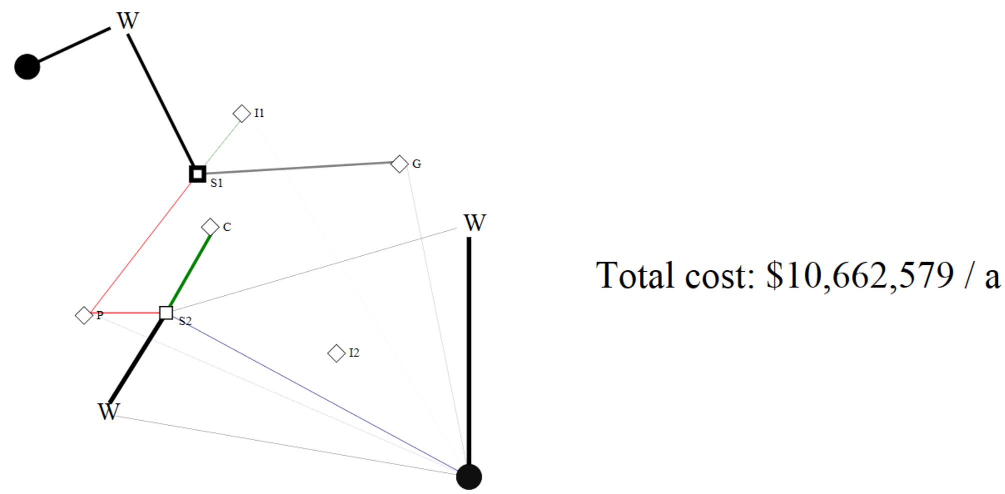


Figure 4: Simulation, $t = 4$

We see that at time $t = 4$, the two landfills have been filled (the circles blacken - see figure 4). As we optimized over a finite period, it is possible that the solution will not be sustainable beyond that period by capacity reasons.

3.2 Dual variables

A great feature of the SIMPLEX algorithm is that it computes optimal dual variables at the same time. Every constraint of the LP gives rise to a dual variable. We can loosely interpret the dual variables as the change in objective function when relaxing the associated constraint.

The most interesting case is to look at the capacity constraints of the facilities.

- A highly negative dual variable for the capacity constraint at facility F identifies F as a bottleneck. Increasing the capacity there will lower the optimal cost of the system considerably. By the complementary slackness theorem, an optimal solution will always exhaust the capacity of F.

- Zero dual variables occur when there is unused (slack) capacity at facilities. We can even consider reducing these in order to save costs when including cost for providing capacity, i.e. wages, building facilities.

Revisiting the graphics in 3, we see that the dual capacity variables are plotted as the thickness of the symbols of the facility. In our case, we see a bottleneck at sorting facility S1. So let's increase the capacity there and re-run the simulation for 5. The optimal waste flow has changed, now giving a new bottleneck at the glass recycling facility G. However, the total annual costs are reduced by almost $4,000,000, now that a lot more waste can be sorted and thus recycled.

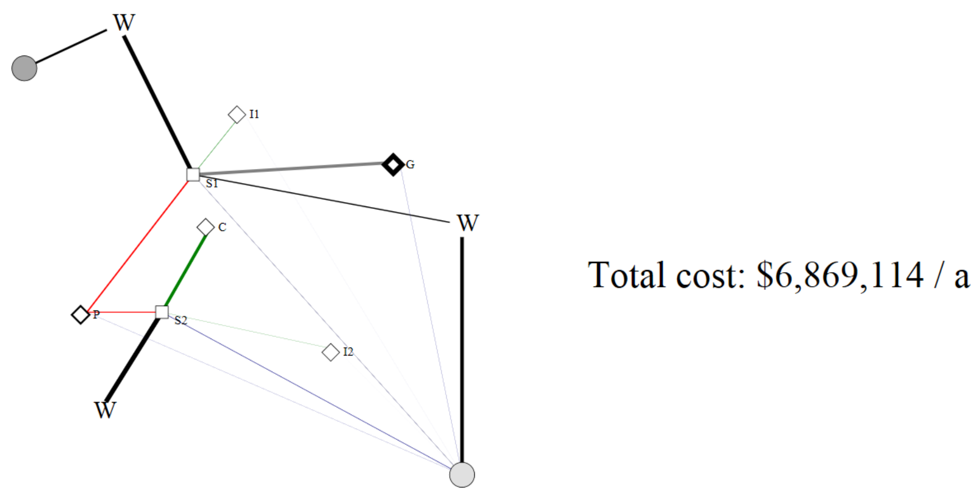


Figure 5: Simulation, $t = 0$

3.3 Optimal capacities

Iterating the above process, we can try to eliminate bottlenecks and determine capacities for all facilities that allow for an optimal waste flow. But that's precisely what the SIMPLEX algorithm does itself, so we could as well add the capacities as LP decision variables to the model. As these occur just as right-hand sides in inequalities, this is barely a change to the program. However for the LP to stay bounded, we have to add a cost for providing capacity per year to the objective function and then minimize the whole system, giving optimal waste flow *and* capacities.

As a comparison in 6, this is the cost that occurred without any recycling.

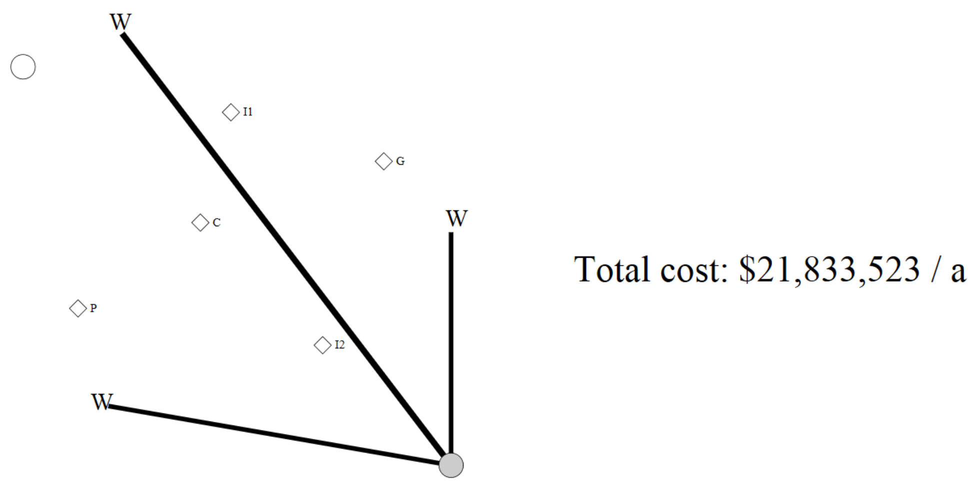


Figure 6: Send everything to the landfills

4 Conclusions

For situations adapted to our model, we are now in a position to answer the questions we have posed at the beginning of the report.

- Is recycling worth it compared to sending everything to landfills?
  Even though this strongly depends on the situation modeled, it can be decided with our model. Furthermore, in our example it turned out that recycling can be a more profitable solution than waste disposement at landfills.

- Can we also obtain profit from unsorted waste sources?
  This is also case-dependent and can be handled using our sorting extension.

- How expensive is recycling if we want to fulfill certain quotas?
  It can be implemented in our model by just adding an extra constraint,

  $$\text{unsorted} \cdot \text{quota} \leq \text{recycled}.$$

- Do capacities represent a bottle-neck for the recycling process?
  Yes, they could, but with our model we can case-study the capacities of the facilities and change them in order to achieve a better performance of the system. This can be done by either looking at the dual variables of the SIMPLEX algorithm or optimizing over the capacities with an adapted cost function.

- Is it feasible to remove or build facilities?
  Also depending on the situation, our model shows which facilities are barely used and we can therefore remove them. On the other hand, if a facility is overburdened we can solve the issue by either increasing its capacity or adding another facility of the same type. Naturally, one has to compare the results to construction and removal costs.

5 Remarks

We would like to remark some improvements or ideas and open problems that we could not investigate given the time available.

- Waste collection is a problem in itself and it can be approached using logistic models.
- We could also optimize over the positions of some facilities in order to further optimize transportation costs, but this would lead to a loss of linearity.
- Open problems and ideas
  - Asymptotic behavior of the flux for $T \to \infty$.
  - Dependence of solutions with respect to initial conditions and model functions could be studied in the field of stability theory.
  - Including transportation time.
  - Continuous time-dependence of the functions.

A Details of the Simulated Model

In the following, we describe the model, which we have actually simulated. In contrast to the theoretical model explained in section 2 all occurring waste is processed completely in one time-step, such that it is not necessary to model storage of waste at processing-facilities.

A.1 Goal

Given the positions of waste sources, processing- and sorting facilities, landfills, processing costs, transportation costs, tipping fees and the amount and composition of waste at each waste source, the aim of the following model is to determine the flow of waste between waste sources, sorting facilities, processing facilities and landfills with respect to capacities such that the total costs are minimal.

A.2 Variables

We will model a planning period T which consists of time-steps t.

A.2.1 Waste Sources

Waste sources are denoted by w. The total amount of waste available in time-step t (e.g. in year t) is denoted as $quantity(w, t)$. We assume that the waste is unsorted but the composition of waste at a source w at time t is known. We consider the following materials m: plastic (P), glass (G), compostable waste (B) and unusable waste (L) which has to be disposed of at a landfill.

We define the following variables:

- $quantity(w, t)$: total amount of waste at source w at time t
- $P(w, t)$: percentage of plastic in the waste from source w at time t
- $G(w, t)$: percentage of glass in the waste from source w at time t
- $B(w, t)$: percentage of compostable waste in the waste from source w at time t

- $L(w, t)$: percentage of unusable waste in the waste from source w at time t

A.2.2 Sorting Facilities

Sorting facilities are denoted by s. Sorting facilities convert unsorted waste into sorted waste. After sorting, components of the waste can be transported separately from other waste. We also consider sorting facilities which do not decompose waste into all of its components (P, G, B, L) but leave some materials unsorted.

Example. A facility $s(FB)(GL)$ decomposes a waste flow into two components. The first component consists of plastic and compostable waste while the second component consist of glass and unusable waste. This facility is useful to decompose waste flows into combustible and non-combustible components.

Each sorting facility s has a capacity $CAPs_t$ in time-step t. If t denotes years this means that s can handle $CAPs_t$ units of waste in year t. The processing costs per unit of (unsorted) waste at time-step t is denoted by Cs_t and is assumed to be independent of the composition of waste.

A.2.3 Processing Facilities and Landfills

Processing facilities are denoted by f. They can handle sorted or partially sorted waste, depending on the facility. Each facility f converts a vector of materials (P, G, B, L) into a new vector of materials by a linear transformation which can be represented by a matrix Pf. Each facility has a capacity $CAPf$ and processing costs. The costs of processing a material vector (P, G, B, L) at time t (in year t) is denoted by $cf_t(P, G, B, L)$. We assume that cf_t is a linear function, so it can be represented by a vector whose entries $costs_{f,t}(m)$ denote the costs to process one unit of material m at facility f at time t (in year t). We may assume that the revenue which a facility gets from selling processed material (recycling facility) is included in the value $costs_{f,t}(m)$.

We consider the following types of facilities

Recycling Facility

A recycling facility f can process plastic P and glass G, where plastic and glass must not be mixed. We assume that the facility can sell all processed material in the same time-step as the processing takes place. The processing matrix Pf is given by

$$Pf = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Hence, glass and plastic are removed while the rest is stored.

Incinerator

An incinerator f can process combustible material. In this model plastic P and compost B can be combusted in the incinerator, where it is not necessary to decompose the waste into plastic and compost before processing but it is necessary that the waste transported to the incinerator is not mixed with non-combustible waste. Let r denote the residues which remain when burning a unit of material m, where $m \in \{P, B\}$. We assume that the residues need to be disposed of at landfills. The processing matrix is given by

$$Pf = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ r & 0 & r & 1 \end{pmatrix}$$

For technical reasons, which will become clear in section A.4.4, we assume that $costs_{I,t}(m) = costs_{\tilde{I},t}(m)$ for all incinerators I and $\tilde{I}$ and all materials m.

Composting Facility

A composting facility f can only handle compostable material B. We assume that the residues of processing B can be sold or disposed of at no costs and therefore do not need to be transported to a landfill. The processing matrix is given by

$$Pf = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

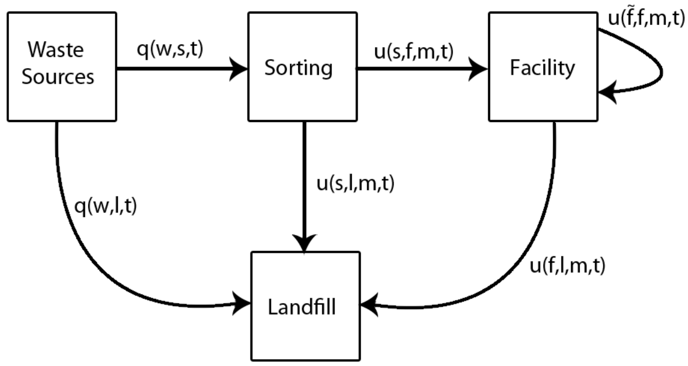
Landfill

A landfill l can handle all sorts of materials. It can handle unsorted and sorted waste streams. Here the costs $costs_{l,t}(m) = costs_{l,t}$ are independent of the material m and correspond to tipping-fees. The processing matrix is a zero-matrix. In contrast to the capacities of the processing facilities, the capacity CAP_l of a landfill l is time independent and CAP_l does *not* describe the capacity in each time-step t but the capacity of l in the whole planning period.

A.2.4 Transport Variables

We consider the following variables

- k_t: transportation costs per unit of waste and unit of distance
- $d_{x,y}$: distance between source/facility/landfill x and source/facility/landfill y
- $q(w, s, t)$: amount of unsorted waste transported from source w to sorting facility s in time-step t
- $q(w, l, t)$: amount of unsorted waste transported from source w to a landfill l in time-step t
- $u(s, f, m, t)$: amount of material m transported from sorting facility s to facility f in time-step t
- $u(s, l, m, t)$: amount of material m transported from sorting facility s to landfill l in time-step t
- $u(\tilde{f}, f, m, t)$: amount of material m transported from processing facility $\tilde{f}$ to facility f in time-step t
- $u(f, l, m, t)$: amount of material m transported from processing facility f to landfill l in time-step t



A.3 Objective Function

The objective function Z is given as the sum of sorting costs, processing costs, tipping fees at landfills and transportation costs.

$$
\begin{aligned}
Z = & \sum_{t,w,s} q(w,s,t) \cdot Cs_t \\
& + \sum_t \left(\sum_{s,f,m} costs_{f,t}(m) \cdot u(s,f,m,t) + \sum_{\tilde{f},f,m} costs_{f,t}(m) \cdot u(\tilde{f},f,m,t) \right) \\
& + \sum_{t,l} cost_{l,t} \cdot \left(\sum_w q(w,l,t) + \sum_{s,m} u(s,l,m,t) + \sum_{f,m} u(f,l,m,t) \right) \\
& + \sum_t k_t \cdot \left(\sum_{w,s} d_{ws} \cdot q(w,s,t) + \sum_{w,l} d_{w,l} \cdot q(w,l,t) \right. \\
& \qquad \left. + \sum_{s,f,m} d_{s,f} \cdot u(s,f,m,t) + \sum_{\tilde{f},f,m} d_{\tilde{f},f} \cdot u(\tilde{f},f,m,t) \right)
\end{aligned}
$$

The aim is to determine all variables of the form $q(x,y,t)$ and $u(x,y,m,t)$ such that Z is minimal. Notice that Z is linear in those variables.

A.4 Constraints

We consider the following constrains.

A.4.1 Waste Sources

The whole amount of waste $quantity(w,t)$ of waste source w in time-step t must be transported to sorting facilities or to landfills in the same time-step. We get the following equation for all t and all waste sources w

$$\sum_s q(w,s,t) + \sum_l q(w,l,t) = quantity(w,t)$$

A.4.2 Mass Balance at Sorting Facilities

Sorting neither creates nor destroys materials. Hence, for all t, all sorting facilities s and all materials m we have

$$\sum_w q(w,s,t) \cdot m(w,t) = \sum_f u(s,f,m,t) + \sum_l u(s,l,m,t),$$

where $m(w,t)$ is the percentage we introduced in A.2.1.

A.4.3 No Storage at Processing Facilities

If a processing function has non-trivial output (e.g. the incinerator generates unusable waste), then the residues must be transported to another processing facility or to a landfill. Therefore for all t and processing facilities f we have the following material vector equality

$$\begin{aligned} \sum_s Pf \begin{pmatrix} u(s,f,P,t) \\ u(s,f,G,t) \\ u(s,f,B,t) \\ u(s,f,L,t) \end{pmatrix} + \sum_{\tilde{f}} Pf \begin{pmatrix} u(\tilde{f},f,P,t) \\ u(\tilde{f},f,G,t) \\ u(\tilde{f},f,B,t) \\ u(\tilde{f},f,L,t) \end{pmatrix} \\ = \quad \sum_l \begin{pmatrix} u(f,l,P,t) \\ u(f,l,G,t) \\ u(f,l,B,t) \\ u(f,l,L,t) \end{pmatrix} + \sum_{\tilde{f}} \begin{pmatrix} u(f,\tilde{f},P,t) \\ u(f,\tilde{f},G,t) \\ u(f,\tilde{f},B,t) \\ u(f,\tilde{f},L,t) \end{pmatrix} \end{aligned}$$

A.4.4 Transportation Constraints

To avoid trivialities we impose that no facility is allowed to send material to itself. For all t, for all materials m and for all facilities f we have

$$u(f, f, m, t) = 0.$$

Furthermore we forbid to transport material to facilities, which cannot handle them. For all t, all sorting facilities s, all materials m and processing facilities $f, \tilde{f}$ such that f cannot handle m we have

$$u(s, f, m, t) = 0$$

$$u(\tilde{f}, f, m, t) = 0.$$

If waste is just partially sorted but not totally decomposed into its components P, G, B, L, we cannot transport each material independently. For instance, assume that we have a sorting facility $s(PB)(GL)$ as in example A.2.2 which decomposes a waste flow into two components (PB) and (GL) consisting of combustible and non-combustible waste. The mixed flow (PB) of plastic and compostable waste can only be processed by incinerators and landfills. The other component (GL) must be disposed off at landfills. We define the following constraints

$$\forall t \, \forall m \in \{P, B\} \, \forall f \neq \text{incinerator} \qquad u(S(PB)(GL), f, m, t) = 0$$

$$\forall t \, \forall m \in \{G, L\} \, \forall f \qquad u(S(PB)(GL), f, m, t) = 0.$$

Now, in principle, the model is still allowed to split materials P and B and send them to different incinerators or send G and L parts to different landfills but the waste is not sorted, so we cannot transport G independently of L. We get around this problem by adapting the interpretation of the variables

$$u(s(PB)(GL), I, P, t), \quad u(s(PB)(GL), I, B, t)$$

and

$$u(s(PB)(GL), l, P, t), \quad (s(PB)(GL), l, B, t)$$

in the following way: We will *not* consider the waste flow between $s(PB)(GL)$ and incinerators, resp. landfills with respect to a single material but only the accumulated flows $u(s(PB)(GL), I, P, t) + u(s(PB)(GL), I, B, t)$ of combustible waste from the sorting facility $s(PB)(GL)$ to incinerators I and the accumulated flows $u(s(PB)(GL), l, G, t) + u(s(PB)(GL), l, L, t)$ of (GL)-waste from the sorting facility $s(PB)(GL)$ to a landfill l.

Example. Consider two incinerators I_1, I_2 and the following material flow

$$\begin{aligned} u(s(PB)(GL), I_1, P, 1) &= 10 \\ u(s(PB)(GL), I_1, B, 1) &= 2 \\ u(s(PB)(GL), I_2, P, 1) &= 5 \\ u(s(PB)(GL), I_2, B, 1) &= 1. \end{aligned}$$

This means we should transport 12 units of combustible waste from the sorting facility to incinerator 1 and 6 units of combustible waste to incinerator 2.

Notice that, although the waste might not be transported in the modeled composition, the total processing costs do not change, if we assume that $costs_{I,t}(m) = costs_{\tilde{I},t}(m)$ for all incinerators I and $\tilde{I}$. Since we assume that the residues of burning material P and material B are identically r, the total transportation costs do not change either.

A.4.5 Capacity Constraints

We use the following inequalities to make sure that the model respects the capacity constraints of the facilities considered.

$$\forall t\, \forall s \qquad \sum_{w} q(w, s, t) \leq CAPs_t$$

We assume that the capacities of a processing facility do not depend on the incoming materials. (It is very easy to relax this assumption.) We get

$$\forall t\, \forall f \qquad \sum_{s,m} u(s, f, m, t) + \sum_{\tilde{f},m} u(\tilde{f}, f, m, t) \leq CAPf_t.$$

To make sure that the capacities of the landfills l are considered we use the following inequality:

$$\forall l \qquad \sum_{t} \left(\sum_{w} q(w, l, t) + \sum_{s,m} u(s, l, m, t) + \sum_{f,m} u(f, l, m, t) \right) \leq CAPl$$

A.5 Extension of our model

We can also determine optimal capacities such that the total costs are minimized. Therefore we optimize over all capacities next to waste flows and add the following term to our objective function:

$$Z_{new} = Z + \sum_{s} CAPs_t \cdot \alpha_{s,t} + \sum_{f} CAPf_t \cdot \alpha_{f,t} + \sum_{l} CAPl \cdot \alpha_l,$$

where $\alpha_{s,t}$ denotes the costs to provide a capacity unit at sorting facility s in time-step t (similar for processing facilities f and landfills l).

We have used a linear solver to solve this optimization problem. See our source code at [2].

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